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D = 5, N = 2 Geometric Higher Curvature Supergravity in the Second-Order Canonical Theory

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The supersymmetric extension of the five-dimensional Chern–Simons gravity is studied from the Hamiltonian point of view. This model containing the Gauss–Bonnet term quadratic in the Riemann curvature is the gauge theory of the supergroup $SU(2, 2/1)$. In the first order, the theory has a polynomial structure, but the second-order leads to a nonpolynomial structure for both the Hamiltonian and the supersymmetry transformation rules of the fields. The second-order theory has the advantage that the apparent gauge degrees of freedom are unambiguously removed leaving only the physical ones. This important feature is analyzed by constructing the second-order Hamiltonian theory. The gauge invariances of the model and the generator of time evolution are found.

KEY WORDS: geometric supergravity model; higher curvature supergravity; canonical theory.

1. INTRODUCTION

Recently, the five-dimensional Chern–Simons pure gravity theory (Macías and Lozano, 2001) was formulated in the framework of the canonical covariant formalism (CCF) (Zandron, 2003a). Later on, having in mind the higher derivative character of the model, the second-order theory was also constructed (Zandron, 2003b). The explicit covariance of the CCF in all their steps makes possible to obtain equation of motion constraints and all the dynamical quantities in a very simple and compact form.

In Zandron (2003a) it was also shown that the relation between the CCF and the usual canonical component theory is not trivial. This was done by means of an integral relationship relating the form brackets introduced in the CCF, with the standard Poisson brackets defined in the canonical component formalism. This connection between these two different treatments is only possible in the first-order

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CCF, when both field variables the fünfbein V^a and the spin connection ω^{ab} are considered as independent variables.

As was shown in Zandron (2003a) for pure $D = 5$ gravity, the second-order Hamiltonian formalism cannot be implemented directly from the CCF due to the higher derivative character of the model. Second time derivatives on the fünfbein and the electromagnetic fields appear when the spin connection is considered as an dependent variable.

Consequently, in the Dirac picture we are in the presence of a constrained Hamiltonian system with a singular higher-order Lagrangian. Therefore, the introduction of the canonical momenta is implemented by means of the Ostrogradski transformation (Kentwell, 1988; Kersten, 1988; Nesterenko, 1989; Nesterenko and Nguyen, 1988; Zi-ping, 1990, 1991a,b).

In the present paper we start by considering the $D = 5$ higher curvature supergravity theory constructed by supersymmetric completion of the Gauss–Bonnet five-bosonic form (Ferrara *et al.*, 1987). The Lagrangian five-bosonic form of this purely geometric model has two pieces, one linear and the other one quadratic in the Riemann curvature. The motivation of the paper is to construct the second-order Hamiltonian theory. As well known the second-order theory is necessary to remove without unambiguity the apparent gauge degrees of freedom from the true physical ones. The first-class constraints which verify the constraint algebra are found, and so all the Hamiltonian gauge symmetries can be constructed. This step is needed when the model is analyzed from the quantum point of view. The paper is organized as follows: In Section 2, the main features of the structure of the $D = 5$ geometric higher curvature supergravity are analyzed. In Section 3 the fundamental results obtained in the context of the CCF are reviewed. In Section 4, the Hamiltonian second-order theory is constructed. Conclusions are given in Section 5.

2. GEOMETRIC HIGHER CURVATURE SUPERGRAVITY IN FIVE SPACE–TIME DIMENSUIONS

From a long time ago geometrical models of supergravity in five space–time dimensions were constructed in the framework of the supergroup manifold approach (D'Auria *et al.*, 1981, 1982). In particular, $N = 2$ geometrical supergravity in five space–time dimensions can be formulated in both the Poincaré and the anti de Sitter versions. In what follows we restrict the attention to the Poincaré version that is the gauge theory of the $\overline{SU(2, 2/1)}$ Inonü-Wigner contraction of the $SU(2, 2/1)$ supergroup.

The curvatures are defined by

$$R^a = \mathcal{D}V^a - \frac{i}{2}\bar{\xi}_M \wedge \Gamma^a \xi_M, \tag{1}$$

$$R^{ab} = d\omega^{ab} - \omega^{ac} \wedge \omega^{cb} \tag{2}$$

$$\rho_M = \mathcal{D}\xi_M = d\xi_M - \frac{1}{4}\omega^{ab} \wedge (\Gamma_{ab}\xi_M) \tag{3}$$

$$R^{\otimes} = dA - i\bar{\xi}_M \wedge \xi_M. \tag{4}$$

where the one-form gauge fields are (ω^{ab}, V^a, ξ_M, A) (spin connection, fünfbein, gravitino and Maxwell field respectively), being ξ_M a pseudo Majorana doublet of spinors: $C\bar{\xi}_M^T = \varepsilon_{MN}\xi_N$. The indices $a, b, c \ldots = 1, 2, 3, 4, 5$ are used in tangent space (Lorentz indices).

In Ref. (Ferrara *et al.*, 1987) by demanding that the theory has interactions with only one time derivative, it has been shown that a Lagrangian density $\mathcal{L}_{(2)}^{(\text{geom})}$ quadratic in the Riemann curvature can be added to the following geometrical Lagrangian density $\mathcal{L}_{(1)}^{(\text{geom})}$ linear in the Riemann curvature,

$$\begin{aligned}
\mathcal{L}_{(1)}^{(\text{geom})} = {} & \frac{1}{3}R^{ab} \wedge V^c \wedge V^d \wedge V^e \varepsilon_{abcde} + \eta R^{ab} \wedge V_a \wedge V_b \wedge A \\
& + i\eta R^a \wedge \bar{\xi}_M \wedge \Gamma_a \xi_M \wedge A \\
& + i(1-\eta)R^a \wedge \bar{\xi}_M \wedge \xi_M \wedge V_a + \frac{i}{2}R^{\otimes} \wedge \bar{\xi}_M \wedge \xi_M \wedge A \\
& - \frac{i}{2}(3+\eta)R^{\otimes} \wedge \bar{\xi}_M \wedge \Gamma_a \xi_M \wedge V^a - \frac{(1+\eta)}{4}\bar{\xi}_M \wedge \xi_M \wedge \bar{\xi}_N \wedge \xi_N \wedge A \\
& + \frac{(1+\eta)}{2}\bar{\xi}_M \wedge \xi_M \wedge \bar{\xi}_N \wedge \Gamma_a \xi_N \wedge V^a + 2i\bar{\xi}_M \wedge \Gamma_{ab}\rho_M \wedge V^a \wedge V^b \\
& + \frac{1}{4}R^{\otimes} \wedge R^{\otimes} \wedge A + \eta R^a \wedge R_a \wedge A,
\end{aligned} \tag{5}$$

previously determined in Castellani *et al.* (1983).

The Lagrangian density $\mathcal{L}_{(2)}^{(\text{geom})}$ is obtained by considering the supersymmetric completion of the Chern–Simons (or Gauss–Bonnet) term $R^{ab} \wedge R^{cd} \wedge V^e \varepsilon_{abcde}$ quadratic in the Riemann curvature. It can be shown that there are only two other terms quadratic in curvatures of scale dimension $w = 1$, and so the Lagrangian density reads

$$\mathcal{L}_{(2)}^{(\text{geom})} = \alpha'(R^{ab} \wedge R^{cd} \wedge V^e \varepsilon_{abcde} + R^{ab} \wedge R_{ab} \wedge A + 2i\bar{\xi}_M \wedge \Gamma_{ab}\rho_M \wedge R^{ab}), \tag{6}$$

where α' is a parameter of scale dimension $w = 2$. We note that by a partial integration the term $R^{ab} \wedge R_{ab} \wedge A$ is written

$$Tr\left(R \wedge \omega + \frac{1}{3}\omega \wedge \omega \wedge \omega\right)(R^{\otimes} + i\bar{\xi}_M \wedge \xi_M), \tag{7}$$

exhibiting the Chern–Simons form of the Lorentz group.

Consequently, the total Lagrangian density $\mathcal{L}$ of the $D = 5$ higher curvature supergravity is given by

$$\mathcal{L} = \mathcal{L}^{(\text{geom})}_{(1)} + \mathcal{L}^{(\text{geom})}_{(2)} \tag{8}$$

In the group manifold approach the rheonomy principle is introduced (Castellani *et al.*, 1983), and it essentially states that the outer components of the curvature remain determined by the inner ones.

In a first-order Lagrangian theory (Ferrara *et al.*, 1987) has been shown that the variational field equation coming from the Lagrangian (8) admits the rheonomic solution, which projected in the inner directions ($4V^a$-projections or space–time equations) imply the following constraints on the inner components

Einstein equation

$$R^{rb}_{rc} - \frac{1}{2}\delta^b_c R^{rs}_{rs} - 960\alpha' \delta^{[brspq]}_{[cijkl]} R^{ij}_{rs} R^{kl}_{pq} = F_{cr} F^{br} - \frac{1}{2}\delta^b_c F_{rs} F^{rs}, \tag{9}$$

gravitino equation

$$\left(\Gamma_{ab} + \alpha' R^{pq}_{ab} \Gamma_{pq}\right) \rho^M_{cd} \varepsilon^{abcde} = 0, \tag{10}$$

Maxwell equation

$$\varepsilon^{abcde} \left[R^{ab}_{cd} + \alpha' R^{ab}_{pq} R^{pq}_{cd} + \frac{1}{2} F_{ab} F_{cd} \right] = 0, \tag{11}$$

torsion equation

$$\varepsilon_{abcde} U^{cd}_{[rs} R^e_{pq]} + U^{ab}_{[rs} F_{pq]} + 2i\alpha' \bar{\rho}^M_{[rs} \Gamma^{ab} \rho^M_{pq]} = 0, \tag{12}$$

where in the last equation, U^{ab}_{rs} is defined by

$$U^{ab}_{rs} = \delta^{ab}_{rs} + 2\alpha' R^{ab}_{rs}. \tag{13}$$

Therefore, the inverse of U, i.e. $(U^{-1})^{rs}_{ab} U^{ab}_{pq} = \delta^{rs}_{pq}$ is given as a power series of the parameter α'

$$(U^{-1})^{rs}_{ab} = \delta^{rs}_{pq} - 2\alpha' R^{rs}_{ab} + 4(\alpha')^2 R^{rs}_{pq} R^{pq}_{ab} + \dots \tag{14}$$

With these definitions the inner-inner component of the torsion R^a is given by

$$R^a_{bc} = -\frac{\eta}{4} \varepsilon^{abcde} F_{de} + 0(\alpha')\text{terms}. \tag{15}$$

The Riemann curvature implicit in (U^{-1}) is given in terms of the spin connection corresponding to the inner-inner component of the torsion R^a. Looking at the Eq. (12) we can see that it is a first-order nonlinear differential equation for the inner-inner components of the torsion, which must be solved iteratively as a power series in α'. So, the solution for the holonomic components of the total spin connection can be formaly written in terms of the fields (V^a_μ, ξ_M) and $F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$ as follows

$$\omega^{ab}_\mu(V, \xi, F) = \widehat{\omega}^{ab}_\mu(V, \xi) + \overline{\omega}^{ab}_\mu(F) + 0(\alpha')\text{terms}, \tag{16}$$

where $\widehat{\omega}^{ab}_\mu(V, \xi)$ is the ordinary spin connection for supergravity in five-dimensions

$$\begin{aligned}\widehat{\omega}^{ab}_\mu(V, \xi) = {}& \frac{1}{2} V^{a\nu}\left(\partial_\mu V^b_\nu - \partial_\nu V^b_\mu\right) - \frac{1}{2} V^{b\nu}\left(\partial_\mu V^a_\nu - \partial_\nu V^a_\mu\right) \\ & - \frac{1}{2} V^{a\rho} V^{b\sigma}(\partial_\rho V_{c\sigma} - \partial_\sigma V_{c\rho}) V^c_\mu \\ & + \frac{\kappa^2}{2}\left(\bar{\xi}_{M\mu}\Gamma^a \xi^b_M - \bar{\xi}_{M\mu}\Gamma^b \xi^a_M + \bar{\xi}^a_M \Gamma_\mu \xi^b_M\right),\end{aligned} \tag{17}$$

and $\overline{\omega}^{ab}_\mu(F)$ writes

$$\overline{\omega}^{ab}_\mu(F) = -\frac{\eta}{4}\varepsilon^{abcde} F_{cd} V_{e\mu} = \frac{\eta}{4}\varepsilon^{abcde}(\partial_\sigma A_\rho - \partial_\rho A_\sigma) V^\sigma_c V^\rho_d V_{e\mu}. \tag{18}$$

In Eqs. (15–16), the term we call $0(\alpha')$terms contains higher derivative expressions in the fünfbein, the gravitino and Maxwell field.

Therefore, when the second-order Hamiltonian theory is implemented and the torsion mechanism of propagation takes place, the nonpolynomic structure is made evident.

3. SUMMARY OF THE MAIN RESULTS OBTAINED IN THE CCF

In the framework of the Hamiltonian theory (Nelson and Regge, 1986) the higher curvature model under consideration requires the application of the extended CCF previously developed (Foussats and Zandron, 1989, 1990). The four Hamiltonian field equations are obtained from the consistency condition on the primary constraints

$$d\phi_\Sigma = (\phi_\Sigma, H_T) = -(\text{Field equations of motion}) + (\phi_\Sigma, Z^\Theta) \wedge \phi_\Theta \approx 0, \tag{19}$$

where the compound indices Σ and Θ take the values $((ab), a, \alpha, \otimes)$.

By straightforward calculation and after some lengthy algebraic manipulations the field equations of motion are written as follows torsion equation

$$\begin{aligned}d\phi_{ab} = {}& -(R^c \wedge V^d \wedge V^e \varepsilon_{abcde} + \eta R^\otimes \wedge V_a \wedge V_b + 2\alpha' R^c \wedge R^{de}\varepsilon_{abcde} \\ & + 2\alpha' R^\otimes \wedge R_{ab} + 2i\alpha' \bar{\rho}_M \wedge \Gamma_{ab}\rho_M) = 0,\end{aligned} \tag{20}$$

Einstein equation

$$
\begin{aligned}
d\phi_a = -(&R^{bc} \wedge V^d \wedge V^e \varepsilon_{abcde} + 2i R^a \wedge \bar{\xi}_M \wedge \xi_M + 2\eta R_a \wedge R^{\otimes} \\
&- \frac{i}{2}(3-\eta) R^{\otimes} \wedge \bar{\xi}_M \wedge \Gamma_a \xi_M - 4i \bar{\xi}_M \wedge \Gamma_{ab} \rho_M \wedge V^b \\
&+ 2i(1-\eta)\bar{\rho}_M \wedge \xi_M \wedge V_a + \alpha' R^{bc} \wedge R^{de} \varepsilon_{abcde}) = 0,
\end{aligned} \tag{21}
$$

gravitino equation

$$
\begin{aligned}
d\phi_M^{\alpha} = -(&4i(\Gamma_{ab}\rho_M)^{\alpha} \wedge V^a \wedge V^b - 4i(\Gamma_{ab}\xi_M)^{\alpha} \wedge R^a \wedge V^b \\
&+ 2i(1-\eta)\xi_M^{\alpha} \wedge R^a \wedge V_a - i(3+\eta)(\Gamma_a \xi_M)^{\alpha} \wedge R^{\otimes} \wedge V^a \\
&+ 2i\alpha'(\Gamma_{ab}\rho_M)^{\alpha} \wedge R^{ab}) = 0,
\end{aligned} \tag{22}
$$

Maxwell equation

$$
\begin{aligned}
d\phi_{\otimes} = -\Bigg(&\eta R^{ab} \wedge V_a \wedge V_b - \frac{i}{2}(3-\eta) R^a \wedge \bar{\xi}_M \wedge \Gamma_a \xi_M + \frac{3}{2} i R^{\otimes} \wedge \bar{\xi}_M \wedge \xi_M \\
&- i(3+\eta)\bar{\rho}_M \wedge \Gamma_a \xi_M \wedge V^a + \eta R^a \wedge R_a \\
&+ \frac{3}{4} R^{\otimes} \wedge R^{\otimes} + \alpha' R^{ab} \wedge R_{ab}\Bigg) = 0.
\end{aligned} \tag{23}
$$

The above motion Eqs. (20–23) have nontrivial solutions only for $\eta = \pm 1$. For all other values of η the solution is given by the vacuum one, i.e $R^a = R^{ab} = \rho = R^{\otimes} = 0$.

In Eq. (19) the four primary constraints ϕ_Σ which determine the total Hamiltonian $H_T = H_{\text{can}} + \Lambda^\Sigma \wedge \phi_\Sigma$ (Foussats and Zandron, 1989) are given by

$$
\phi_a = \pi_a - \omega^{bc} \wedge V^d \wedge V^e \varepsilon_{abcde} - i(1-\eta)\bar{\xi}_M \wedge \xi_M \wedge V_a - \eta \Lambda^{\otimes} \wedge V_a \approx 0, \tag{24}
$$

$$
\begin{aligned}
\phi_{ab} = \pi_{ab} - 2\alpha'[&(\Lambda^{cd} + C^{cd}) \wedge V^e \varepsilon_{abcde} + (\Lambda_{ab} + C_{ab}) \wedge A \\
&+ i\bar{\xi}_M \wedge \Gamma_{ab}(\Lambda_M + C_M)] \approx 0,
\end{aligned} \tag{25}
$$

$$
\phi_M^{\alpha} = \pi_M^{\alpha} - 2i(\Gamma_{ab}\,\xi_M)^{\alpha} \wedge V^a \wedge V^b - 2i\alpha'(\Gamma_{ab}\,\xi_M)^{\alpha} \wedge (\Lambda^{ab} + C^{ab}) \approx 0, \tag{26}
$$

$$
\begin{aligned}
\phi_{\otimes} = \pi_{\otimes} &- \eta\omega^{ab} \wedge V_a \wedge V_b + \frac{i}{2}(3+\eta)\bar{\xi}_M \wedge \Gamma_a \xi_M \wedge V^a \\
&- \eta \Lambda^a \wedge V_a - \frac{1}{2}\Lambda^{\otimes} \wedge A \approx 0.
\end{aligned} \tag{27}
$$

where we call $\Lambda^\Sigma = d\mu^\Sigma$ and the two-forms C^Σ with constant coefficients are defined by

$$
C^a = -\omega^{ac} \wedge V_c - \frac{i}{2}\bar{\xi}_M \wedge \Gamma^a \xi_M, \tag{28}
$$

$$C^{ab} = -\omega^{ac} \wedge \omega^{cb}, \tag{29}$$

$$C_M = -\frac{1}{4}\omega^{ab} \wedge (\Gamma_{ab}\xi_M), \tag{30}$$

$$C^{\otimes} = -i\bar{\xi}_M \wedge \xi_M. \tag{31}$$

When the form-brackets $(\phi_\Sigma, \phi_\Theta)$ between constraints are explicitly computed, it is easy to show that none of the primary constraints ϕ_Σ is first class. The general result is that in the CCF there are not first-class constraints. On the other hand, the primary constraints we have found in the CCF, in the framework of the second-order canonical theory, will be no longer relationships between field and momentum. These relations depending on the velocities cease to be constraints. By looking at the Eqs. (24–27) it can be seen that the primary constraints (up to first order in α') are written

$$\phi_\Sigma = \phi_\Sigma^o + \alpha' \phi'_\Sigma, \tag{32}$$

where the piece ϕ'_Σ in general will contain curvatures.

The Eq. (32) can even be treated as a constraint by considering that there are two versions of the CCF (Foussats and Zandron, 1990) and briefly these are

(i) The version valid for supergravities described by a linear Lagrangian in curvatures, in which the independent fields variables are the potentials μ^Σ.
(ii) The extended version valid for supergravities described by a general polynomial Lagrangian in curvatures, in which the independent field variables are μ^Σ and Λ^Σ.

In both versions the total Hamiltonian is given by the same expression, i.e., by the equation $H_T = H_{\text{can}} + \Lambda^\Sigma \wedge \phi_\Sigma$. The relation between both versions can be obtained only by means of the introduction of the Dirac's brackets. Once this is done we can reconsider that $\Lambda^\Sigma = \Lambda^\Sigma(\mu)$ are arbitrary polynomials in the field variables μ^Σ with nonconstant coefficients satisfying the Bianchi identities $d\Lambda^\Sigma = 0$. Precisely, this procedure yields that the Eq. (32) turns out to be dependent on the velocities, losing the constraint condition.

Consequently, when the second-order theory is implemented, the new primary constraints must be determined.

Finally, the motion field equations (four-forms) (20–23) can be projected along the different sectors $V^3\xi$, $V^2\xi^2$, $V\xi^3$ and ξ^4. Thus, it can be shown how the outer components of the different curvatures can be determined in terms of the inner ones. That is, the motion field equations are rheonomics (Castellani *et al.*, 1983; D'Auria *et al.*, 1981, 1982).

4. SECOND-ORDER CANONICAL THEORY AND NEW CONSTRAINTS

In order to derive the second-order canonical theory the model must be arranged (Dirac, 1962; Nelson and Teitelboim, 1977, 1978; Teitelboim, 1977). As it was shown in Zandron (2003b) it is not possible go over to the second-order theory directly from the CCF because of the higher-derivative character of the model. Therefore, we must return to the original Lagrangian (8) and it must be written in components.

To construct the second-order canonical theory the first step is to carry out the space–time decomposition in the manifold M^5 as done in Zandron (2003b) (see also Seahra and Wesson, 2003). Next, all the equations and quantities given in form language must be written in components. All the dynamical fields must be considered as reduced forms, i.e. forms defined on the physical space M^5. Moreover, we assume that the reduced forms defined on M^5 are written in the holonomic basis dx^μ. Therefore, equations, fields and forms must be projected on a space-like $x^0 = t = t^0$ hypersurface Σ of four dimensions. This is done by considering the injection map $\chi : \Sigma \to M^5$ in such a way that the associated pull-back χ^* acts on any generic form by setting $t = t^0$ and $dt = 0$.

Let us use the space–time decomposition given in Section 2 of Zandron (2003b). So, from now on we use Greek indices $\mu, \nu, \rho, \ldots = 0, 1, 2, 3, 4$ for space time tensors (world indices) and Latin indices $i, j, k, \ldots$ for label spatial components only. The alternating tensors $\varepsilon_{i_1 \ldots i_4}$ on the hypersurface Σ and $\varepsilon_{0,i_1 \ldots i_4}$ on the manifold M^5 are related by the equation: $N^\perp \varepsilon_{i_1 \ldots i_4} = -\varepsilon_{0,i_1 \ldots i_4}$. Moreover, $\varepsilon_{\mu_1 \ldots \mu_5} = V_{a_1 \mu_1} \ldots V_{a_5 \mu_5} \varepsilon^{a_1 \ldots a_5}$. In the explicit computations the above relations and a set of formulae given in Nelson and Regge (1986) relating the fünfbein and the alternating tensors on Σ, on the space–time M^5, and on the corresponding tangent space are systematically used.

Following Zandron (2003b), the five-dimensional metric tensor $g^{(5)}_{\mu\nu}$ splits according to $N^\perp = (-g^{00})^{1/2}$, $N_i = g_{0i}$, $g = det(g^{(4)})$, $(-g^{(5)})^{1/2} = N^\perp g^{1/2}$. Moreover, an arbitrary five-dimensional vector $\mathcal{V}^a$ can be decomposed as follows: $\mathcal{V}^a = \mathcal{V}^\perp n^a + \mathcal{V}^i V^a_i$, where $\mathcal{V}^\perp = -\mathcal{V}_\perp = -n_a \mathcal{V}^a$ and $\mathcal{V}_i = \mathcal{V}^a V_{ai}$.

In the second-order CFF the metricity condition on both the manifold M^5 and the four-dimensional hypersurface Σ can be considered. The first metricity condition or fünfbein postulate implicates that "the fünfbein is covariantly constant." That is the full covariant derivative including both the spin and the world (metric) connection satisfies the standard metricity condition

$$\partial_\mu V^a_{.\nu} + \omega^{ab}_\mu V_{b\nu} - \Gamma^{(5)\rho}_{.\mu\nu} V^a_{.\rho} = 0, \tag{33}$$

where $\Gamma^{(5)\rho}_{.\mu\nu}$ is the affine connection on the five-dimensional manifold M^5.

By considering the spin connection Ω^{ab} and the affine connection $\Gamma^{(4)i}_{.jk}$ on the hypersurface Σ, we also have

$$\partial_k V^a_{.j} + \Omega^{ab}_k V_{bj} - \Gamma^{(4)i}_{.kj} V^a_{.i} = 0, \tag{34}$$

by virtue of the metricity condition on the four dimensional hypersurface Σ.

Moreover, by multiplying the Eq. (34) by n_a holds

$$\partial_k n^a + \Omega^{ab}_k n_b = 0. \tag{35}$$

On the other hand, in the Eqs. (33) and (34) both affine connections $\Gamma^{(5)\rho}_{.\mu\nu}$ and $\Gamma^{(4)i}_{.jk}$ are not longer symmetric in their lower indices due to the torsion generated by the spinor field ξ_M. Moreover, a fundamental difference between the Eqs. (33) and (34) is given by the fact that the affine connection $\Gamma^{(4)i}_{.jk}$ as functional of the metric $g^{(4)}_{ij}$ on the hipersurface Σ, only depends on V_{ai}. So, the spin connection Ω^{ab}_k is a functional of V_{ai} and its spatial derivatives only. On the other hand, the restriction to Σ of the affine connection $\Gamma^{(5)\rho}_{.\mu\nu}$, i.e $\Gamma^{(5)i}_{.jk}$ and ω^{ab}_k not only depends of V_{ai} but also of their conjugate momenta.

Now, by considering the restriction to the hypersurface Σ of the Eq. (1) rewritten in components, and the explicit expression (15) for $\alpha' = 0$, the inner–inner components of the torsion can be written

$$\partial_{[k} V^a_{.j]} + \omega^{ab}_{[k} V_{bj]} - \frac{i}{2}\bar{\xi}_{M[k}\Gamma^a \xi_{Mj]} - \frac{\eta}{4}\varepsilon^{abcde} V_{bk} V_{cj} F_{de} = 0. \tag{36}$$

Taking into account Eqs. (33) and (36) the following expression for the torsion S^ρ_{kj} holds

$$S^\rho_{kj} \equiv \frac{1}{2}\left(\Gamma^{(5)\rho}_{.kj} - \Gamma^{(5)\rho}_{.jk}\right) = \frac{i}{2}\bar{\xi}_{M[k}\Gamma^\rho \xi_{Mj]} + \frac{\eta}{4} V^\rho_a \varepsilon^{abcde} V_{bk} V_{cj} F_{de} \tag{37}$$

As it can be seen from the Eqs. (33) and (34), in the second-order CFF both spin connections ω^{ab}_μ and Ω^{ab}_i, in five and four dimensions respectively, can be determined completely. After some algebraic manipulations the well known relationship between the spatial components of both spin connections are found

$$\omega^{ab}_i = \Omega^{ab}_i + (n^b V^{aj} - n^a V^{bj}) K_{ij}, \tag{38}$$

where K_{ij} is the extrinsic curvature on the four-dimensional surface Σ in the manifold M^5. The extrinsic curvature tensor K_{ij} is defined by the following general equation

$$K_{ij} = \frac{1}{N^\perp}(-\dot{g}_{ij} + N_{i\|j} + N_{j\|i}) - C_{ij\perp}, \tag{39}$$

where the double stroke $\|$ denotes the covariant derivative on the four-surface Σ only including the affine connection.

The components $C_{\rho\mu\nu}$ of the contorsion tensor in the holonomic basis at zero order in the parameter α' are defined by

$$C_{\rho\mu\nu} = -(S_{\mu\rho\nu} + S_{\nu\mu\rho} + S_{\rho\mu\nu}), \tag{40}$$

and so the components $C_{\rho\mu\nu}$ and $C_{\rho\mu\nu}$ are respectively given by

$$C_{ij\perp} = -\frac{i}{2}(\bar{\xi}_{Mi}\Gamma_j\xi_{M\perp} - \bar{\xi}_{Mi}\Gamma_\perp\xi_{Mj} + \bar{\xi}_{Mj}\Gamma_i\xi_{M\perp}) - \frac{\eta}{4}\varepsilon_{ijkl}F^{kl} \tag{41}$$

$$C_{ijk} = -\frac{i}{2}(\bar{\xi}_{Mi}\Gamma_j\xi_{Mk} - \bar{\xi}_{Mi}\Gamma_k\xi_{Mj} + \bar{\xi}_{Mj}\Gamma_i\xi_{Mk}) + \frac{\eta}{4}\varepsilon_{ijkl}F^{l\perp}. \tag{42}$$

Moreover, for the component $\omega_\perp^{\cdot ab}$ the following equation holds

$$N^\perp\omega_\perp^{ab} = \frac{1}{2}\big(V^{ak}\delta_N V_k^b - n^a\delta_N n^b - [a \to b]\big) - (V^{ak}n^b - V^{bk}n^a)\partial_k N^\perp, \tag{43}$$

where

$$\delta_N V^{ak} = \partial_0 V^{ak} - \mathcal{L}_{N^k}V^{ak}, \tag{44}$$

$$\delta_N n^a = \partial_0 n^a - \mathcal{L}_{N^k}n^a, \tag{45}$$

and $\mathcal{L}_{N^k}$ stands for the Lie derivative operator along N^k in the four-dimensional hypersurface Σ.

We note that the components of the contorsion tensor (40) admits an expansion in power series of the parameter α' analogously to those given in Eqs. (15) and (16) for the inner-inner components of the torsion and the spin connection respectively.

At this stage, the Lagrangian (8) must be written in components. We only write explicitly the piece $\mathcal{L}_{(2)}^{(\text{geom})}$ of this Lagragian in which second-time derivatives on the fünfbein and the electromagnetic fields appear once the spin connection is eliminated as independent variable.

So, the Lagrangian $\mathcal{L}_{(2)}^{(\text{geom})}$ without considering total exterior derivatives reads

$$\begin{aligned}\mathcal{L}_{(2)}^{(\text{geom})} = N^\perp g^{1/2}\varepsilon^{\alpha\mu\nu\rho\sigma}\alpha'\Big\{&\varepsilon_{abcde}\big[\partial_\alpha\omega_\mu^{de}\big(\omega_\nu^{bc}\omega_\rho^{af}V_{\sigma f} + 2\omega_\nu^{bf}\omega_\rho^{fc}V_\sigma^a\big)\\ &- \omega_\alpha^{bf}\omega_\mu^{fc}\omega_\nu^{dg}\omega_\rho^{ge}V_\sigma^a\big] + \partial_\alpha\omega_\mu^{ab}(\omega_{\nu ab}\partial_\rho A_\sigma - 2\omega_{\nu af}\omega_{\rho fb}A_\sigma)\\ &+ \omega_\alpha^{af}\omega_\mu^{fb}\omega_{\nu ag}\omega_{\rho gb}A_\sigma\\ &+ 2i\left[\bar{\xi}_{M\alpha}(\Gamma_{ab}\partial_\mu\xi_{M\nu}) - \frac{1}{4}\omega_\mu^{cd}(\bar{\xi}_{M\alpha}\Gamma_{ab})(\Gamma_{cd}\xi_{M\nu})\right]R_{\rho\sigma}^{ab}\Big\},\end{aligned} \tag{46}$$

Analogously, it can be written in components the piece $\mathcal{L}_{(1)}^{(\text{geom})}$ of the total Lagrangian density (8).

From (46) it can be seen that the second-time derivatives appear because in the Chern–Simons form it is not possible their elimination by partial integration. Consequently, in the framework of the Dirac theory we are in presence of a constrained Hamiltonian system with a singular higher-order Lagrangian.

We start by defining the following independent dynamical field variables

$$V_{a\mu} = (V_{ai}; V_{a0} = n_a N^{\perp} + N^i V_{ai}), \tag{47}$$

$$B_{a\mu} = \partial_0 V_{a\mu}, \tag{48}$$

$$A_{\mu} = (A_i; A_0), \tag{49}$$

$$C_{\mu} = \partial_0 A_{\mu}, \tag{50}$$

$$\xi_{M\mu} = (\xi_{Mi}; \xi_{Mo}). \tag{51}$$

Consequently, by means of the Ostrogradski transformation (Nesterenko, 1989; Zi-ping, 1990, 1991a,b) the following canonical momenta are well defined

$$\Pi^{\mu}_{M}(\xi) = \frac{\partial \mathcal{L}}{\partial(\partial_0 \bar{\xi}_{M\mu})}, \tag{52}$$

$$\Pi^{(1)\mu}_{a} = \frac{\partial \mathcal{L}}{\partial B^{a}_{\mu}} - \partial_{\nu}\left[\frac{\partial \mathcal{L}}{\partial(\partial_{\nu} B^{a}_{\mu})}\right], \tag{53}$$

$$\Pi^{(2)\mu}_{a} = \frac{\partial \mathcal{L}}{\partial\left(\partial_0 B^{a}_{\mu}\right)}, \tag{54}$$

$$\pi^{(1)\mu} = \frac{\partial \mathcal{L}}{\partial C_{\mu}} - \partial_{\nu}\left[\frac{\partial \mathcal{L}}{\partial(\partial_{\nu} C_{\mu})}\right], \tag{55}$$

$$\pi^{(2)\mu} = \frac{\partial \mathcal{L}}{\partial(\partial_0 C_{\mu})}. \tag{56}$$

The Poisson brackets for pairs of canonical conjugate variables are given by

$$\left[\bar{\xi}^{(\alpha)}_{M\mu}(x), \Pi^{\nu}_{N(\beta)}(y)\right] = \left[\Pi^{\nu}_{N(\beta)}(y), \bar{\xi}^{(\alpha)}_{M\mu}(x)\right] = \delta^{(\alpha)}_{(\beta)}\delta^{\nu}_{\mu}\delta_{MN}\delta(x-y), \tag{57}$$

$$\left[V^{a}_{\nu}(x), \Pi^{(1)\mu}_{b}(y)\right] = -\left[\Pi^{(1)\mu}_{b}(y), V^{a}_{\nu}(x)\right] = \delta^{a}_{b}\delta^{\mu}_{\nu}\delta(x-y), \tag{58}$$

$$\left[B^{a}_{\nu}(x), \Pi^{(2)\mu}_{b}(y)\right] = -\left[\Pi^{(2)\mu}_{b}(y), B^{a}_{\nu}(x)\right] = \delta^{a}_{b}\delta^{\mu}_{\nu}\delta(x-y), \tag{59}$$

$$\left[A_{\nu}(x), \pi^{(1)\mu}(y)\right] = -\left[\pi^{(1)\mu}(y), A_{\nu}(x)\right] = \delta^{\mu}_{\nu}\delta(x-y), \tag{60}$$

$$\left[C_{\nu}(x), \pi^{(2)\mu}(y)\right] = -\left[\pi^{(2)\mu}(y), C_{\nu}(x)\right] = \delta^{\mu}_{\nu}\delta(x-y). \tag{61}$$

Having in mind the total Lagrangian (8) written in components from the Equation (52–56) and by means of straightforward but heavy algebraic manipulation the different momenta can be computed and the following results are found:

(a) When the spatial components of the momentum (52) and (54), i.e $\Pi_c^{(1)i}$and $\pi^{(1)i}$ respectively are explicitly computed expressions depending on the velocities are obtained, therefore these relations do not define constraints.
(b) The remaining components of the momenta defined by (52–56) are relationships between fields and canonical conjugate momenta independent on the velocities, given rise the primary constraints in the second-order theory.

The fermionic primary constraints can be written as follows

$$\begin{aligned}\Psi_M^p(\xi) = \Pi_M^\mu(\xi) - \alpha'\big(\Gamma^a\xi_M^b - \Gamma^b\xi_M^a\big)\mathcal{Q}_{abjkl}\big(\omega_{jab}, V_k^a, A_k\big)\varepsilon^{pjkl}\\ - \alpha'\big(V^{ap}\Gamma_i\xi_M^b - V^{bp}\Gamma_i\xi_M^a\big)\mathcal{Q}_{abjkl}\big(\omega_{jab}, V_k^a, A_k\big)\varepsilon^{ijkl}\\ - 2i\left[\frac{1}{2}\big(V_k^aV_l^b - V_k^bV_l^a\big) + \alpha' R_{kl}^{ab}\right]\varepsilon^{pjkl} \approx 0,\end{aligned} \tag{62}$$

$$\Psi_M^0(\xi) = \Pi_M^0(\xi) - \alpha'\big(V^{a0}\Gamma_i\xi_M^b - V^{b0}\Gamma_i\xi_M^a\big)\mathcal{Q}_{abjkl}\big(\omega_{jab}, V_k^a, A_k\big)\varepsilon^{ijkl} \approx 0, \tag{63}$$

where the bosonic functional $\mathcal{Q}_{abjkl}(\omega_{jab}, V_k^a, A_k)$ reads

$$\mathcal{Q}_{abjkl}\big(\omega_{jab}, V_k^a, A_k\big) = N^\perp g^{1/2}\big[\omega_{jab}(\partial_k A_l) - 2\omega_{jac}\omega_{kb}^c A_l + \omega_j^{cd}(\partial_k V_l^e)\varepsilon_{abcde}\big]. \tag{64}$$

Analogously, it can be computed the remaining bosonic primary constraints. They can be formally written as follows

$$\Phi_c^{(2)i} = \Pi_c^{(2)i} - \mathcal{F}_c^i\big(\omega_k^{;ab}, V_\mu^a, A_\mu, \xi_{M\mu}\big) \approx 0, \tag{65}$$

$$\Phi_c^{(2)0} = \Pi_c^{(2)0} \approx 0, \tag{66}$$

$$\phi^{(2)i} = \pi^{(2)i} - \mathcal{G}^i\big(\omega_k^{;ab}, V_\mu^a, A_\mu, \xi_{M\mu}\big) \approx 0, \tag{67}$$

$$\phi^{(2)0} = \pi^{(2)0} \approx 0, \tag{68}$$

$$\Phi_c^{(1)0} = \Pi_c^{(1)0} - \mathcal{J}_c^0\big(\omega_k^{;ab}, V_\mu^a, A_\mu, \xi_{M\mu}\big) \approx 0, \tag{69}$$

$$\phi^{(1)0} = \pi^{(1)0} - \mathcal{K}^0\big(\omega_k^{;ab}, V_\mu^a, A_\mu, \xi_{M\mu}\big) \approx 0. \tag{70}$$

The above functions we are named $\mathcal{F}_c^i$, $\mathcal{G}^i$, $\mathcal{J}_c^0$ and $\mathcal{K}^0$ are expressions which depend on the spatial components and the perpendicular component of the total spin connection (16) and its spatial derivatives, as well as the components of the remaining fields. By means of lengthy but direct calculations, it is possible to show that the constraints and the canonical Hamiltonian $\mathcal{H}_{\text{can}}$ can be written in terms of the canonical momenta and the quantities V_{ai}, A_μ, $\xi^\perp_M$, ξ^i_M, Ω^{ab}_i, $\omega^{\cdot ab}_\perp$, K_{ij}, N_i and $N^\perp$. It is not necessary to give explicitly the expressions of the above functions since our final purpose is to write the total Hamiltonian generator of time evolution in terms of the first-class constraints.

From the momenta given in (52–56), the canonical Hamiltonian $\mathcal{H}_{\text{can}}$ remains defined by

$$\mathcal{H}_{\text{can}} = B^a_\mu \Pi^{(1)\mu}_a + \dot{B}^a_\mu \Pi^{(2)\mu}_a + C_\mu \pi^{(1)\mu} + \dot{C}_\mu \pi^{(2)\mu} + \dot{\bar{\xi}}_{M\mu} \Pi^\mu_M(\xi) - \mathcal{L} \tag{71}$$

where it was replaced $\partial_0 V^a_\mu$ for B^a_μ and $\partial_0 A_\mu$ for C_μ . We note that the canonical Hamiltonian is formed by eliminating only the velocities $\partial_0 B^a_\mu$ and $\partial_0 C_\mu$. The field B^a_μ and C_μ cannot be eliminated from the theory when we treat with higher derivative Lagrangians (Zandron, 2003b). Once the explicit expression of the $\mathcal{L}$ is used in the expression of $\mathcal{H}_{\text{can}}$, the velocities $\dot{B}^a_\mu$ and $\dot{C}_\mu$ are eliminated.

Finally, the total Hamiltonian generator of time evolution of generic functionals is given by

$$H_T = \int d^4x \mathcal{H}_T, \tag{72}$$

where

$$\mathcal{H}_T = \mathcal{H}_{\text{can}} + \lambda^{(2)c}_{.\mu} \Phi^{(2)\mu}_c + \lambda^{(1)c}_{.0} \Phi^{(1)0}_c + \chi^{(2)c}_{.\mu} \Phi^{(2)\mu}_c + \chi^{(1)c}_{.0} \Phi^{(1)0}_c + \theta_{M\mu} \Psi^\mu_M. \tag{73}$$

In Eq. (73) the arbitrary bosonic and fermionic Lagrange multipliers are evaluated by means of the Hamilton equations $\dot{A} = [A, H_T]_{\text{PB}}$.

At this stage, from the stationary condition on the primary constraints, it is possible to define successively the secondary constraints according to the well known Dirac algorithm

$$\Omega^{(k)} = \left[\Omega^{(k-1)}, H_T\right]_{\text{PB}}. \tag{74}$$

This algorithm must be continued until $\Omega^{(k)}$ satisfies

$$\Omega^{(k+1)} = \left[\Omega^{(k)}, H_T\right]_{\text{PB}} = C^a_{.cn} \Omega^{(n)}_a. \tag{75}$$

It can be shown that in the model under consideration there is a set of secondary constraints. By explicit computation it can be shown that

$$\Omega^{(1)}_c = \dot{\Phi}^{(2)0}_c = \left[\Phi^{(2)0}_c, H_T\right]_{\text{PB}} \approx 0, \tag{76}$$

and

$$\Omega^{(1)} = \dot{\phi}^{(2)0} = [\phi^{(2)0}, H_T]_{\text{PB}} \approx 0, \tag{77}$$

are weakly zero quantities.

From now on, following the Dirac's prescriptions, the procedure can be continued for each one of the constraints. The Poisson brackets different from zero which must be evaluated are essentially $[\Pi^{(2)i}_c(x), \omega^{ab}_\mu(y)]_{\text{PB}}$, $[\Pi^{(1)\rho}_c(x), \omega^{ab}_\mu(y)]_{\text{PB}}$,

$[\pi^{(2)i}(x), \omega_\mu^{ab}(y)]_{\text{PB}}$, $[\pi^{(1)\mu}(x), \omega_\mu^{ab}(y)]_{\text{PB}}$ and $[\Pi^{M\rho}(x), \omega_\mu^{ab}(y)]_{\text{PB}}$. Though, the explicit computation is straightforward, it involves heavy algebraic manipulations.

Moreover, when the computation of the Poisson brackets is carried out, it can be seen that none of the secondary constraints is first class.

Some conclusions can be obtained. Looking at the primary constraints (65–70) and taking into account the secondary constraints constructed by means of application of the Dirac algorithm (74), it can be seen that the uniques primary constraints having vanishing Poisson brackets with all the other ones are $\Phi_c^{(2)0}$ and $\phi^{(2)0}$. So, the primary constraints $\Phi_c^{(2)0}$ and $\phi^{(2)0}$ are first-class and they correspond to gauge invariances of the model under local gauge transformations.

Consequently, it can be said that the five-dimensional higher curvature supergravity theory in the second-order theory has primary and secondary constraints. This set has constraints of first and second class. The presence of second-class constraints make necessary to follow the prescriptions of the Dirac theory. In this sense, the Dirac brackets must be first defined from the Poisson brackets, and next the second-class constraints must be eliminated from the theory by taking them strongly equal to zero.

On the other hand, by assuming for simplicity that $\alpha' = 0$, it is possible to show that the other first-class constraints can be constructed by considering appropriate linear combination of constraints. It is well known that these 4-form antisymmetric weakly vanishing quantities $M_{ab}d^4x$, remain defined by

$$M_{ab}d^4x = \Psi_a \wedge V_b - \Psi_b \wedge V_a \approx 0. \tag{78}$$

These 10 constraints M_{ab} are the generators of the local Lorentz group for all the fields of the model (Foussats *et al.*, 1992; Nelson and Regge, 1986).

In Eq. (78) $\Psi_a = \phi_a \mid_\Sigma$ (restriction to Σ of the primary constraint (24)), is the unique constraint that is maintained as weakly zero one, even in the second order theory. Precisely, the reason is that a suitable linear combination of it generates a first-class constraint. Contrarily to what it happens in the canonical component theory (Castellani *et al.*, 1982; Deser and Isham, 1976; Pilati, 1978) this fact naturally appears in the CCF.

By following Nelson and Teitelboim (1977, 1978) the momentum three-form Π_a in the second-order theory is defined by

$$\Pi_a = \pi_a - \Omega^{de} \wedge V^b \wedge V^c \varepsilon_{abcde} \tag{79}$$

where Ω^{de} was defined in (38). After straightforward calculation the constraint Ψ_a is written

$$\Psi_a = \Pi_a + 4\left(V_a^k g^{(4)ij} - V_a^i g^{(4)jk}\right) K_{kj} \Sigma_i - 4n_a C^i_{jk} g^{(4)jk} \Sigma_i. \tag{80}$$

Finally, in order to write the generator of time evolution in its final form the total Hamiltonian H_T (72) is written as follows

$$\begin{aligned} H_T &= \int \mu_0^\Sigma \mathcal{H}_\Sigma(x) d^4x \\ &= \int \left[\frac{1}{2}\omega_0^{ab}\mathcal{H}_{ab}(x) + V_0^a \mathcal{H}_a(x) + \bar{\xi}_{M0}\mathcal{H}_M(x) + \bar{\mathcal{H}}_M(x)\xi_{M0} + A_0 \mathcal{H}_\otimes(x) \right], \end{aligned} \tag{81}$$

where we have written explicitly the Lagrange multipliers μ_0^Σ given by the time components of the field variables $\mu^\Sigma = (\omega^{ab}, V^a, \xi_M, A)$. By following usual methods, it can be seen that the weakly zero functions $\mathcal{H}_{ab}(x)$, $\mathcal{H}_a(x)$, $\mathcal{H}_M(x)$ and $\mathcal{H}_\otimes(x)$ are precisely the projections on the hypersurface Σ of the motion Eqs. (20–23) (torsion, Einstein, gravitino, and Maxwell equations respectively), plus weakly zero quantities (Foussats *et al.*, 1992; Nelson and Teitelboim, 1977, 1978; Teitelboim, 1977). By starting from Eqs. (20–23) after some algebraic manipulations and by neglecting total divergences, it is possible to arrive at the following expressions for the four-forms $\mathcal{H}_\Sigma(x)d^4x$

$$\begin{aligned} \mathcal{H}_{ab}(x)d^4x &= -[R^c \wedge V^d \wedge V^e \varepsilon_{abcde} + \eta R^\otimes \wedge V_a \wedge V_b] \,|_\Sigma \\ &\quad + (\Psi_a \wedge V_b - \Psi_b \wedge V_a) \approx 0, \end{aligned} \tag{82}$$

$$\begin{aligned} \mathcal{H}_a(x)d^4x &= -[R^{bc} \wedge V^d \wedge V^e \varepsilon_{abcde} + 2iR^a \wedge \bar{\xi}_M \wedge \xi_M + 2\eta R_a \wedge R^\otimes \\ &\quad - \frac{i}{2}(3-\eta)R^\otimes \wedge \bar{\xi}_M \wedge \Gamma_a \xi_M - 4i\bar{\xi}_M \wedge \Gamma_{ab}\mathcal{D}\xi_M \wedge V^b \\ &\quad + 2i(1-\eta)\mathcal{D}\bar{\xi}_M \wedge \xi_M \wedge V_a]|_\Sigma - 2\left(\frac{\eta}{4}\varepsilon^{cabde}F_{de}V_b + \omega^{ca}\right) \wedge \Psi_c \approx 0, \end{aligned} \tag{83}$$

$$\begin{aligned} \mathcal{H}_M(x)d^4x &= -[4i(\Gamma_{ab}\mathcal{D}\xi_M) \wedge V^a \wedge V^b - 4i(\Gamma_{ab}\xi_M) \wedge R^a \wedge V^b \\ &\quad + 2i(1-\eta)\xi_M \wedge R^a \wedge V_a - i(3+\eta)(\Gamma_a \xi_M) \wedge R^\otimes \wedge V^a] \,|_\Sigma \\ &\quad - \frac{i}{2}\xi_M \Gamma^a \wedge \Psi_a \approx 0, \end{aligned} \tag{84}$$

$$\begin{aligned} \mathcal{H}_\otimes(x)d^4x &= -\left[\eta R^{ab} \wedge V_a \wedge V_b - \frac{i}{2}(3-\eta)R^a \wedge \bar{\xi}_M \wedge \Gamma_a \xi_M + \frac{3}{2}iR^\otimes \wedge \bar{\xi}_M \wedge \xi_M \right. \\ &\quad \left. - i(3+\eta)\mathcal{D}\bar{\xi}_M \wedge \Gamma_a \xi_M \wedge V^a + \eta R^a \wedge R_a + \frac{3}{4}R^\otimes \wedge R^\otimes \right] |_\Sigma = 0. \end{aligned} \tag{85}$$

By straightforward computation it is possible to show that the weakly zero quantities $\mathcal{H}_\Sigma(x)$ defined in Eqs. (82–85) are the generators of all the Hamiltonian gauge symmetries. Moreover, this set of first-class constraints closes the constraint superalgebra, in complete analogy with what happened in the simple supergravity case (Nelson and Teitelboim, 1977, 1978), that is

$$[\mathcal{H}_{\Sigma}(x)\,,\;\mathcal{H}_{\Theta}(y)] = \Lambda^{\Xi}_{\Sigma\Theta}\mathcal{H}_{\Xi}(x)\delta(x-y), \tag{86}$$

where $\Lambda^{\Xi}_{\Sigma\Theta} = R^{\Xi}_{\Sigma\Theta} - C^{\Xi}_{\Sigma\Theta}$ (for the compound indices Σ,Θ,Ξ) are the *structure functions* for curvatures $R^{\Xi}_{\Sigma\Theta}$ and structure constant $C^{\Xi}_{\Sigma\Theta}$ of the correspondent graded Lie algebra.

5. CONCLUSIONS

Recently (Zandron, 2003a), the topological five-dimensional Chern–Simons pure gravity was formulated in the framework of the first-order extended canonical covariant theory (CCF). The relation between the CCF and the usual first-order canonical formalism written in components was also given. This relation was done by means of a nontrivial integral relationship between the form brackets and the usual Poisson brackets.

As was shown, the CCF is not a proper canonical theory because it does not propagate data defined on an initial hypersurface as required by a standard mechanical system.

In spite of this, at classical level the CCF is a powerful method to understand the structure of the gravitational field, particularly in more than four dimensions and for higher curvature gravity models. The CCF is covariant in all its steps because of the use of exterior algebra. This allows to find the equations of motion and the constraints in a very simple way without introducing complicate calculations.

Moreover, from the CCF only is possible go over to the proper canonical formalism in the first-order formulation, i.e. when the spin connection, the fünfbein and the electromagnetic field are taken as independent dynamical variables.

Contrarily to what it happens in the CCF, in the second-order CFF after long and heavy algebraic manipulations, cumbersome noncovariant expressions for the physical quantities are obtained.

As it was shown, the torsion equation allows to obtain the second-order canonical formalism starting from the first-order one. In the Riemannian pure gravity case, the torsion equation $R^a = 0$ must be considered as an strongly equal to zero constraint, and so the spin connection is solved in terms of the fünfbein.

On the other hand, in D = 5 higher curvature supergravity the first-order CCF contains a finite number of terms in the Hamiltonian. The torsion equation of motion (12) is a first-order differential equation, which must be solved *via* an iterative procedure given rise a power series solution in the curvature. Thus, when the mechanism of the torsion equation is used, we arrive to the second-order formalism in which the nonpolynomial structure of the theory is made evident.

Because of the higher-derivative character of the model, the presence of second-time derivatives on the fünfbein and on the electromagnetic fields, makes necessary the implementation of the Ostrogradski transformation in order to introduce canonical momenta. Essentially, this implicates to take the first-time derivate of both the fünfbein and the electromagnetic fields as independent dynamical ones.

Once the space–time decomposition in M^5 was performed, the constrained Hamiltonian system must be treated as usual according to the Dirac prescriptions. The canonical Hamiltonian is evaluated from the Eq. (71). Later on, the total Hamiltonian (72) as generator of time evolution can be given in terms of the first-class constraints (82–85) which close the constraint superalgebra (86). Therefore, all the Hamiltonian gauge symmetries remain determined and so, the apparent gauge degrees of freedom can be unambiguously removed leaving only the physical ones.

It is important to note that starting from the CCF the first-class constraints are directly obtained by restricting to the hypersurface Σ the field equations of motion arising from the CCF. In particular the first-class constraint $\mathcal{H}_{ab}$ (associated to the Lagrange multiplier ω_o^{ab}) naturally appears. It is not necessary to incorporate it by adding by hand terms to the Hamiltonian as usually is done when the starting point is the component formalism (Castellani *et al.*, 1982; Deser and Isham, 1976; Pilati, 1978).

When the model is considered from the quantum point of view the second order formalism, and the knowledge of the constraints (82–85) is unavoidable.

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